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# Fixed-Mobile Substitution and Termination Rates

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# Fixed-Mobile Substitution and Termination Rates\*

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## Abstract

This paper studies the effect of termination rates on substitution between fixed and mobile calls and access, in a model where heterogeneous consumers can subscribe to one or both types of offers. Simulations show that each (fixed or mobile) termination rate has a positive effect on the take-up of the corresponding service, via the waterbed effect, and lowers subscriptions to the other service, via a cost effect. The prevailing asymmetric regulation, with very low fixed and higher mobile termination rates, corresponds to the social optimum. However, the interests of the mobile operators and of the different customer groups do not coincide.

*Keywords:* Network competition; fixed-mobile substitution; termination rates.

*JEL codes:* L51, L92.

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# 1 Introduction

**The Issues at hand.** New technologies and innovative service providers often challenge the provision of traditional services. One of the most striking examples is the advent of mobile telephony, and its impact on fixed telephony services. Only three decades ago, fixed-line operators fulfilled all communications needs over their networks, but in the last 20 years mobile telephony has significantly altered the historical market structure. All over the world, wireless services have recorded substantial growth in terms of subscriber-ship, revenues, and usage, and despite the recent economic crisis, the number of mobile subscribers has continued to grow.

Much of the success of the mobile sector is due to fixed-mobile substitution. Major technological advances and cost reductions have enabled mobile carriers to decrease the difference between fixed and mobile prices, allowing them to become strong competitors to traditional fixed providers. At the same time, the fixed and mobile markets have been subject to regulatory intervention, but to different degrees: While fixed telephony operators' retail and wholesale prices (i.e., the "mobile termination rates" charged to other operators for receiving their calls) tended to be strongly regulated at cost, for mobile operators only termination rates were eventually regulated, and until recently at values far above marginal cost.

Empirical studies have attempted to quantify fixed-mobile substitution (see, for example, the survey by Vogelsang (2010), as well as the following section), but there is a lack of theoretical investigation about the effect of consumers' preference for mobility on fixed-mobile substitution. This paper tries to fill this gap. Clearly, there is a variety of factors that could be considered as potential causes of fixed-mobile substitution, and none on its own provides a sufficient explanation. This paper focuses on the role of asymmetric regulation of termination rates on fixed and mobile networks as a contributing factor.

The paper presents a model with three independent but interconnected operators, one fixed and two mobile networks. The key novelty is the construction of the demand side. All subscribers spend the same fraction of their time outside home, but depending on their personal preference for mobile telephony, they choose whether to subscribe to a fixed or a mobile network, or to both. Differently from the previous literature, this paper considers the only realistic case, where mobile-only, fixed-only and "fixed-mobile" customers are all simultaneously present in market equilibrium. This framework is in line

with recent evidence from the market. The association of European Telecom Regulators (BEREC, 2011) reported that although the share of mobile-only households increased and that of fixed-only households decreased over time, having both is now the most common situation in Europe (62% of households, on average) and that this share is not declining over time.<sup>1</sup>

The model set out below shows that the possibility of call and access substitution can affect call pricing if subscribers are heterogeneous and firms want to price discriminate. More specifically, when mobile termination rates are relatively high, mobile-to-fixed prices are distorted downward from marginal cost, while when mobile termination rates are relatively low (almost at cost) both mobile-to-mobile and mobile-to-fixed calls are downward distorted. Since mobile-only customers both make and receive more calls on their mobile phones than those that hold both a fixed and a mobile subscription, while both types of consumers are equally affected by the level of fixed fees, a usage-discriminating "waterbed effect" arises that transmits termination profits via discounts in call prices.

The main aim of this paper, though, is to analyze the impact of varying levels of termination rates on consumers' subscription decisions. The key question is whether the different regulatory treatment of termination on fixed and mobile networks affects the development of fixed and mobile subscription decisions. Numerical simulations based on the model show that each (fixed or mobile) termination rate has a positive effect on the take-up of the corresponding service, via the waterbed effect, and lowers subscriptions to the other service, via a cost effect. Moreover, because of the same effect higher termination rates increase the number of consumers who do not subscribe to any service.

Finally, the socially optimal combination of termination rates is analyzed: Fixed termination rates should be set at cost, and mobile termination rates above cost. The interests of mobile operators and different consumer groups are not coincident, though. Both mobile operators and mobile-only consumers prefer a fixed termination rate at cost, while users with both fixed and mobile phones and fixed-only consumers prefer a higher level. On the other hand, consumers prefer a level of mobile termination rate at or below

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<sup>1</sup>Based on the 2011 E-communications household survey, the number of households having at least one mobile subscription is rather high and homogeneous – from 82% to 96% (with an average of 89%) – across Europe. On the other hand, fixed-line penetration is extremely heterogeneous: It is very high in countries such as Sweden (98%), the Netherlands (89%) and France (87%), whereas only 17% of Czech households are connected. Mobile-only households range from 2% to 81%, while the share of fixed-mobile users range from 15% to 94%.

the social optimum, while mobile operators would like to set this rate at a higher level.

Summing up, the model implies that high mobile and low fixed termination rates can lead to an additional shift of subscriptions from fixed to mobile networks. Furthermore, taking account of the social benefit from mobility, this structure of termination rates tended to be socially optimal, while regulation of termination rates as such was also justified.

**Literature Review.** There exists a sizeable economic literature on the relationship between fixed and mobile telephony, and on the role of fixed-to-mobile termination rates.

Wright (2002) considers fixed-to-mobile calls with a focus on mobile termination rates, while others (e.g., Valletti and Houppis, 2005) analyze how socially optimal mobile termination rates depend on the magnitude of network externalities, the intensity of competition in the mobile sector, and the distribution of customer preferences. These papers however do not consider the role of fixed-mobile substitution (both at access and service level), and more importantly, they do not take into account consumer heterogeneity with respect to the benefits from mobility, as done in this paper.

Armstrong and Wright (2009), Baake and Mitusch (2009) and Hausman (2012) analyze the role of call substitution and discuss voluntary vs. regulated setting of termination rates. These authors show that substitution between fixed- and mobile-originated calls weakens the competitive bottleneck of call termination, and brings the termination rates that firms would choose closer to the efficient level while remaining above cost. Hence, the welfare gains from regulating mobile termination rates are smaller, while private incentives do not imply excessively high levels. This paper departs from previous studies by analyzing the more realistic situation where substitution affects not only the types of calls made but also consumers' access decisions. Moreover, the current model highlights how subscription decisions and welfare are affected by both mobile and fixed termination rates.

Closest to this paper is Hansen (2006), who also investigates fixed-mobile access substitution in a model with competition in the mobile market and subscribers with varying mobility benefits. Contrary to the present analysis, though, in his model the heterogeneity in the preference for mobility does not affect the types of calls that consumers make. By contrast, this paper assumes that when a consumer is on the road, he cannot

make fixed calls. Moreover, Hansen does not analyze the most realistic and thus most practically relevant market structure that this paper focuses on, with the simultaneous presence of mobile-only, fixed-only, and fixed-mobile subscribers. User mobility is also present in Valletti (2003), but in a model with mobile-only consumers moving between urban and rural areas while obtaining access to mobile services conditional on the mobile operators' coverage.

Though applied to a different setting, the game-theoretic structure of this paper is similar to the one developed by de Bijl and Peitz (2009), who analyze the effect of access and retail price regulation on the adoption of voice over IP telephony. In their model, as in this analysis, consumers first choose which technology to adopt. Firms then compete in their respective markets, given these adoption decisions.

On the policy side, Bomsel et al. (2003) study the impact of fixed and mobile termination rates on fixed-to-mobile or mobile-to-fixed traffic. The authors estimate that the transfer from fixed networks and their customers, as a result of high mobile termination rates, has amounted to 19 billion Euros in France, Germany and the UK over 1998 to 2002. According to these authors, this transfer harmed fixed customers and operators, probably damaged competition in the fixed market, and distorted competition between fixed and mobile operators. The present theoretical analysis confirms that higher mobile termination rates are likely to have increased the adoption of mobile telephony and to have reduced fixed access, but also highlights that this termination rate structure is likely to have increased social welfare when the benefits from mobility are accounted for.

The current paper is theoretical, but it is important to review also the relevant empirical evidence on fixed-mobile substitution (see the surveys by Woroch, 2002, and Vogelsang, 2010). The evidence for fixed-mobile substitution is rather mixed but suggests that call and access substitution are increasing over time. For example, Rodini et al. (2003), using data from the US, show that households' subscriptions to second fixed lines and mobile services are access substitutes, while Ward and Woroch (2005, 2010) find substantial substitutability between fixed and mobile subscriptions. Some recent studies provide stronger evidence on fixed-mobile call substitution. Briglauer et al. (2011) find for Austria that fixed and mobile calls are strong substitutes while access substitution is rather weak. Ward and Zheng (2012), using panel data from China, find that fixed and mobile telephony services have become fairly strong substitutes for both usage and

subscriptions. Barth and Heimeshoff (2012a,b), using panel data on EU27 countries, show modest substitution effects on fixed and mobile subscriptions, while the estimated cross-price elasticity of the mobile price on fixed line call demand is relatively large compared to previous studies. Grzybowski (2012) analyzes substitution between access to fixed-line and mobile telephony in the European Union. He finds that decreasing prices for mobile services increases the share of mobile-only households and decreases the share of fixed-only and fixed-mobile households, which suggests substitution between fixed-line and mobile connections. Finally, Grzybowski and Verboven (2013) also analyze the substitution between fixed-line and mobile telecommunications services. The authors show that fixed and mobile connections are generally perceived in the EU as substitutes, especially in regions with a higher GDP per capita, but also that there is much heterogeneity across countries due to the interplay of specific social and economic features (i.e., households with different age, education, professional activity, etc.). Moreover, the authors show that from the firms' point of view there is strong complementarity between fixed-line and mobile connections, enabling incumbents to leverage their strong position in the fixed-line market into the mobile market.

Note that none of these empirical studies presents evidence on the impact of fixed and mobile termination rates on consumers' subscription decisions. Hence, this paper also provides new testable predictions for future empirical work on fixed-mobile substitution.

The paper is organized as follows. In Section 2 the model is presented, and the equilibrium retail tariffs of mobile and fixed operators is found. In Section 3 the effect of fixed and mobile termination rates on subscription decisions and welfare is analyzed. Finally, in Section 4 conclusions are reported. Proofs can be found in the Appendix.

## 2 Model and Pricing Equilibrium

### 2.1 Setup

**Consumers and firms.** There are three firms: two mobile networks and one monopoly fixed network.<sup>2</sup> The fixed network is regulated; its call prices are set by the regulator at marginal costs, and its total profits are equal to zero.

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<sup>2</sup>The fixed network is independent of the mobile networks, that is, fixed-mobile integration is not considered. See Hoernig et al. (2014) for an analysis of this issue.

Consumers, whose total mass is fixed at 1, can subscribe to a mobile network and/or to the fixed network. According to their network choice, they are denoted mobile-only (M), fixed-only (F), and fixed-mobile (FM) subscribers. Consumers spend a fraction of their time  $\lambda \in (0, 1)$  outside home. When they are on the move, which happens with probability  $\lambda$ , consumers can only use their mobile phones to make or receive calls. When they are at home, which happens with probability  $1 - \lambda$ , consumers have access to a fixed and/or a mobile phone; if they have both, they choose whichever is cheaper to make calls.

Calling patterns are balanced, and consumers are on the move or at home independently of each other.<sup>3</sup> They are heterogenous in their relative benefits from mobility and, independently, in their preferences for the two mobile networks, as specified below.

It is further assumed that when consumers choose which network(s) to join, they know their benefits from mobility, but do not yet know their network preference. For example, consumers may first decide whether or not to have a mobile phone and which one (e.g., an iPhone or an Android phone), and only afterwards decide which mobile operator to subscribe to.<sup>4</sup>

Mobile network  $i$ 's numbers of M- and FM-clients are  $\mu_i^m$  and  $\mu_i^{mx}$ , respectively, for  $i = 1, 2$ . Its total number of subscribers is then  $\mu_i = \mu_i^m + \mu_i^{mx}$ . The total numbers of M- and FM-clients are  $\mu^m = \mu_1^m + \mu_2^m$  and  $\mu^{mx} = \mu_1^{mx} + \mu_2^{mx}$ , respectively, the total number of mobile subscribers is  $\mu = \mu^m + \mu^{mx}$ , and there are  $\mu^x$  fixed clients. Consumers and firms are assumed to have rational expectations about the number of subscribers for each network technology ( $\mu^m$ ,  $\mu^{mx}$  and  $\mu^x$ ). That is, they take these as given when they make their subscription and tariff decisions, but their value must be consistent *ex post* with equilibrium tariffs and subscriber numbers.<sup>5</sup> On the other hand, firms take into account that their market shares  $\mu_i^m$  and  $\mu_i^{mx}$  are affected directly by tariff decisions.<sup>6</sup>

Finally, mobile networks incur a fixed cost  $f$  per subscriber, marginal costs of origination and termination  $c_o$  and  $c_t$ , and on-net costs  $c = c_o + c_t$ . The mobile termination rate,

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<sup>3</sup>Intertemporal substitution, i.e. the possibility for consumers that are on the road to postpone calls until they arrive at home, is ignored.

<sup>4</sup>This timing assumption leads to simpler expressions for market shares and keeps the analysis technically feasible.

<sup>5</sup>Thus the determination of customer segments corresponds to the "passive belief" approach suggested by Hurkens and Lopez (2014).

<sup>6</sup>A model where  $\mu^m$ ,  $\mu^{mx}$  and  $\mu^x$  vary with out-of-equilibrium tariffs (rather than only equilibrium tariffs) has been also analyzed by the authors. The results are qualitatively similar though mathematically much more complicated.



applied regardless of whether calls originate from a mobile or a fixed network, is  $a \geq c_t$ . Denote by  $n = a - c_t$  the termination mark-up. Similarly, the fixed network has a fixed cost per subscriber  $f_x$ , marginal costs  $c_{xo} < c_o$  and  $c_{xt} < c_t$ , on-net costs  $c_x = c_{xo} + c_{xt}$ , and a termination rate  $a_x \geq c_{xt}$  with mark-up  $n_x = a_x - c_{xt}$ . Given that fixed termination rates tend to be regulated close to cost, it is assumed throughout that  $a_x < c_t$ .

For now, the following assumption regarding the marginal costs of calls is made:

**Assumption 1:** The marginal costs of calls, including termination rates, are ordered as follows:

$$c_x < c_o + a_x < c_o + \frac{c_t + a}{2} < c_{xo} + a. \quad (1)$$

Given that marginal costs and termination rates on fixed networks tend to be far below those of mobile networks, the only actually restrictive assumption is the right-most inequality. It states that the mobile termination rate is so high that fixed-to-mobile calls have higher perceived marginal costs than mobile-to-mobile calls, that is,  $a > a^* \equiv c_t + 2(c_o - c_{xo})$ . This assumption describes an initial phase of development of mobile markets, where mobile termination rates are much higher than fixed termination rates. For example, a recent report by BEREC (2013) shows that the average termination rate in the EU27 in January 2013 was equal to 2.58€cents/minute, while the average fixed termination rate ranged between 0.50 and 0.80€cents/minute according to the level (Layer) of interconnection. Hence, termination on mobile networks is still at least three times more expensive than on fixed networks.

However, in many EU countries termination rates are still being cut considerably by national regulators. This implies that in the medium-term termination rates will approach marginal cost (i.e.,  $n \rightarrow 0$ ). Hence, the ordering of marginal costs of calls may change. Therefore, the case of very low mobile termination rates is also considered below.

**Tariffs and surplus.** Mobile network  $i$  charges a tariff  $(F_i, p_i, p_{ix})$ , where  $F_i$  is a monthly fixed fee, and  $p_i$  and  $p_{ix}$  are the mobile-to-mobile and mobile-to-fixed per-minute call prices.<sup>7</sup> Similarly, the fixed network offers a tariff  $(F_x, p_x, p_{xm})$ , where  $F_x$  is its monthly fixed fee,  $p_x$  its on-net price, and  $p_{xm}$  the fixed-to-mobile per-minute call price. It is postulated (and confirmed in the simulations below), that the following ordering of

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<sup>7</sup>In order to focus on the effects of interconnection between fixed and mobile networks, a uniform mobile-to-mobile call price is assumed, ruling out different prices for on-net and off-net calls.

prices holds:

$$p_x < p_{ix} < p_i < p_{xm}. \quad (2)$$

Note that due to assumption (1), condition (2) holds when equilibrium prices are equal or close to their respective marginal cost.

The consumption utility from a call of length  $q$  is  $u(q)$ , where  $u' > 0$  and  $u'' < 0$ . For  $k \in \{x, ix, i, xm\}$ , the caller's indirect utility is  $v_k = v(p_k) = \max_q u(q) - p_k q$ , and the call duration is  $q_k = -v'(p_k)$ . Apart from the surplus obtained from making calls, subscribers obtain an access surplus that depends on the network(s) they subscribe to. If a consumer subscribes only to a mobile network or to the fixed network, his subscription surplus is  $A_m$  or  $A_x$ , respectively. If he subscribes to both types of networks, he obtain an access surplus of  $A_{mx}$ .

When an FM-subscriber of network  $i$  is on the move, the order of prices (2) implies that it is cheaper for him to make mobile-to-fixed calls than mobile-to-mobile calls when receivers are at home ( $p_{ix} < p_i$ ). Thus, mobile-to-mobile calls are only made when receivers are themselves on the move.<sup>8</sup> When at home, the same subscriber can use either his fixed phone or his mobile phone. From (2), he uses his fixed phone to call other people at home ( $p_x < p_{ix}$ ), and his mobile phone to call people on the move ( $p_i < p_{xm}$ ). The expected surplus of subscribing both to the fixed network and to mobile network  $i$  is thus given by

$$w_i^{mx} = A_{mx} - F_i - F_x + \rho^m v_i + \rho^x [\lambda v_{ix} + (1 - \lambda) v_x], \quad (3)$$

where  $\rho^m = \mu^m + \lambda \mu^{mx}$  denotes the number of receivers on mobile phones, i.e. all M-subscribers plus FM-subscribers when the latter are on the move. Equally,  $\rho^x = (1 - \lambda)(\mu^{mx} + \mu^x)$  describes the number of receivers on fixed phones, which are all F- and FM-subscribers if and only if they are at home.

The surplus (3) is equal to the access surplus minus the fixed fees, plus the surplus of calling M-subscribers and FM-subscribers on the move, plus the surplus of calling FM- and F-subscribers when they are at home, either from the road with probability  $\lambda$  or from home with probability  $1 - \lambda$ . At home, there is substitution to cheaper fixed on-net

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<sup>8</sup>This assumption describes the procedure in which the caller first calls the receiver's fixed line and then calls him on his mobile if he does not find him at home.

calls to reach fixed phones and to cheaper mobile-to-mobile calls to reach mobile phones (fixed-to-mobile calls are never used by FM-subscribers).

A mobile-only user makes all calls with his mobile phone, and obtains the surplus

$$w_i^m = A_m - F_i + \rho^m v_i + \rho^x v_{ix}. \quad (4)$$

Similarly, a fixed-only user makes all calls from his fixed line, and receives the surplus

$$w^x = A_x - F_x + (1 - \lambda) (\rho^m v_{xm} + \rho^x v_x). \quad (5)$$

Note that fixed-only users are the only group of consumers that makes fixed-to-mobile calls.

**Subscription decisions and market shares.** As already mentioned, consumers differ in their preferences for mobile networks. They learn these preferences after deciding to take out a mobile subscription, but before choosing a specific mobile network. The former decision is based on the expected surplus of having a mobile subscription, while the latter decision is taken after comparing actual tariffs and takes into account the preference for one or the other network.

For given tariffs  $(F_i, p_i, p_{ix})$  and resulting surpluses  $w_i^m$  and  $w_i^{mx}$ , and  $j \neq i$ , mobile operator  $i$ 's market shares of M- and FM-subscribers are assumed to be

$$\mu_i^m = y(w_i^m - w_j^m) \mu^m, \text{ and } \mu_i^{mx} = y(w_i^{mx} - w_j^{mx}) \mu^{mx},$$

respectively, where  $y$  is a function  $y : \mathbb{R} \rightarrow [0, 1]$ , with  $y(0) = 1/2$ ,  $y' > 0$  and  $y'(0) = \sigma > 0$ . An often-used special case is the Hotelling model, which over the relevant range corresponds to  $y(x) = 1/2 + \sigma x$ .

Consumers are assumed to correctly anticipate that the mobile market equilibrium will be symmetric; they therefore expect to subscribe to each mobile network with probability  $1/2$ . *Ex post*, consumers choose the mobile network that is closer to their taste. Denoting by  $\kappa > 0$  the expected disutility from not obtaining a perfect match in their operator

choice,<sup>9</sup> subscribers expect surplus  $\bar{w}^x$  on the fixed network, and

$$\bar{w}^m = \frac{1}{2}(w_1^m + w_2^m) - \kappa, \text{ and } \bar{w}^{mx} = \frac{1}{2}(w_1^{mx} + w_2^{mx}) - \kappa.$$

Since consumers have rational expectations, in equilibrium it is imposed that  $\bar{w}^k = w^k$ , for  $k \in \{m, mx, x\}$ , that is, the expected surpluses must be equal to the realized ones.

Consumer heterogeneity is captured in the benefits from mobility with the following assumption: The utility of consumer  $l$  from taking subscription decision  $k \in \{m, mx, x\}$  is defined by the linear random utility model

$$U_{lk} = \bar{w}^k + \varepsilon_{lk},$$

where  $\varepsilon_{lk}$  is a random term.<sup>10</sup> The number of consumers in each segment  $k \in \{m, mx, x\}$  is then given by an expression

$$\mu^k = P_k(\bar{w}^m, \bar{w}^{mx}, \bar{w}^x), \quad (6)$$

where  $P_k$  is the probability of choice  $k$ ,  $\partial P_k / \partial \bar{w}^k > 0$ ,  $\partial P_k / \partial \bar{w}^l \leq 0$  for  $l \neq k$ , and  $P_m + P_{mx} + P_x \leq 1$ .

**Profits.** The number of customers of mobile network  $i = 1, 2$  without access to a fixed phone is  $\rho_i^m = \mu_i^m + \lambda \mu_i^{mx}$ , and its profits are given by ( $j \neq i$ )

$$\begin{aligned} \pi_i = & \mu_i \{ F_i - f + [\rho^m(p_i - c) - \rho_j^m n] q_i \} \\ & + \rho_i^m \rho^x (p_{ix} - c_o - a_x) q_{ix} \\ & + \rho_i^m n [\mu_j q_j + (1 - \lambda) \mu^x q_{xm}]. \end{aligned} \quad (7)$$

The first line of equation (7) gives the profits due to fixed fees and calls to other mobile users, the second line the profits from calls to the fixed network, and the last line those from termination.

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<sup>9</sup>For the Hotelling framework, we have  $\kappa = 2 \int_0^{1/2} x / (2\sigma) dx = 1/(8\sigma)$ .

<sup>10</sup>The distribution of  $\varepsilon_{lk}$  is not relevant for the determination of equilibrium tariffs. In our simulations below we assume a logit choice model. The value of any potential outside option is normalized to zero.

For the fixed network, profits are

$$\begin{aligned}
\pi_x = & (\mu^x + \mu^{mx})(F_x - f_x) + (\rho^x)^2(p_x - c_x)q_x \\
& + (1 - \lambda)\mu^x\rho^m(p_{xm} - c_{xo} - a)q_{xm} \\
& + \rho^x n_x(\rho_1^m q_{1x} + \rho_2^m q_{2x}).
\end{aligned} \tag{8}$$

Again, subscription profits are on the first line, together with profits from on-net calls. Profits from calls to mobiles are on the second line, and termination profits are on the last line.

Expected consumer surplus is given by an expression that depends on expected surpluses and the discrete choice model for customer segments,

$$CS = S(\bar{w}^m, \bar{w}^{mx}, \bar{w}^x),$$

and total welfare is

$$W = CS + \pi_1 + \pi_2 + \pi_x. \tag{9}$$

Let us now recall the sequence of events. In a first stage, consumers decide which technology to adopt (F, M, or FM). Then, mobile operators set their retail tariffs. Third, consumers learn their fixed benefits from mobility, and the consumers who adopted the mobile technology choose their mobile operator. Fourth, and finally, consumers make call decisions. The solution concept adopted is subgame-perfect equilibrium.

Consumers' decisions at Stage 3 and 4 have already been determined. The following analysis starts with Stage 2 where mobile operators set their retail tariffs.

## 2.2 Equilibrium Tariffs

The symmetric equilibrium in the mobile market is found by solving mobile networks' profit-maximization problems, taking the segmentation of customers in F-, FM- and M-subscribers as given. On the other hand, the standard procedure of maximizing networks' profits over call prices, while holding the number of each network's subscribers constant, fails because of "composition effects": Mobile-to-fixed prices enter  $\mu_i^m$  and  $\mu_i^{mx}$  with different relative weights. Thus, adjusting the fixed fee  $F_i$  to hold, say,  $\mu_i^m$  constant after a change in  $p_{ix}$ , does not annul the changes in  $\mu_i^{mx}$ . The correct procedure, as applied

in the proof of the following result, is to maximize simultaneously over the whole tariff  $(F_i, p_i, p_{ix})$ .

**Proposition 1** *Under Assumption 1, symmetric mobile equilibrium tariffs are given by:*

1. *A mobile-to-mobile call price equal to average marginal cost,*

$$p_i^* = c + \frac{n}{2}; \quad (10)$$

2. *a mobile-to-fixed call price at or below marginal cost,*

$$p_{ix}^* = c_o + a_x + n (q_{ix}/q'_{ix}) \Omega^{high}, \quad (11)$$

where

$$\Omega^{high} = \frac{\mu q_i + (1 - \lambda) \mu^x q_{xm}}{1 / (2\sigma \Phi) - \rho^x q_{ix}^2 / q'_{ix}}, \quad \Phi = \frac{\mu^m + \lambda^2 \mu^{mx}}{\rho^m} - \frac{\rho^m}{\mu} > 0;$$

3. *fixed fees equal to*

$$F_i^* = f + \frac{1}{2\sigma} \left( 1 - n \frac{\rho^m \Omega^{high}}{\mu \Phi} \right) + \frac{\rho^m}{2} n q_i.$$

4. *Equilibrium profits are*

$$\pi_i^* = \frac{\mu}{4\sigma}.$$

Profits  $\pi_i^*$  and mobile-to-mobile call prices  $p_i^*$  have the standard form for uniform tariffs in the Hotelling model, that is, they are equal to the "transport cost"  $1/2\sigma$  times the subscriber number, and perceived average marginal cost, respectively (Armstrong, 1998; Laffont, Rey and Tirole, 1998a).

The mobile-to-fixed call price  $p_{ix}$  is set below marginal cost if  $n > 0$ . This pricing structure arises because mobile-only customers both make more mobile-to-fixed calls and receive more incoming calls. Thus, when the mobile termination rate is above cost these customers bring in higher termination profits. Competition then transforms the latter into a discount on the service that mobile-only customers use more than other customers, that is, mobile-to-fixed calls. This is borne out by the fact that the distortion in  $p_{ix}$  disappears when the mobile termination rate is set at cost, i.e.,  $a = c_t$ . Thus, the *combination of call substitution and customer heterogeneity* gives rise to a distorted usage pricing structure.

For the regulated monopoly fixed network, cost-based call pricing and the zero-profit constraint imply, given the symmetric equilibrium in the mobile market, that

$$p_x^* = c_x, p_{xm}^* = c_{xo} + a, \text{ and } F_x^* = f_x - (1 - \lambda) n_x \rho^m q_{ix}. \quad (12)$$

### 2.3 Equilibrium Tariffs when Mobile Termination Rates are (almost) at Cost

Suppose now that the mobile termination rate is low, i.e.,  $a \leq a^*$ . The ranking of marginal cost changes to:

**Assumption 2:** The marginal costs of calls are ordered as follows:

$$c_x < c_o + a_x; \quad c_{xo} + a \leq c_o + \frac{c_t + a}{2}. \quad (13)$$

Under this assumption, the relevant ordering of prices becomes:

$$p_x < p_{ix} < p_i; \quad p_x < p_{xm} < p_i.$$

That is, fixed-to-mobile calls become cheaper than mobile-to-mobile calls.<sup>11</sup>

While mobile-only and fixed-only users' surplus remains unchanged, FM-users now can call mobiles from home at a cheaper price. Therefore, their surplus changes to:

$$w_i^{mx} = A_{mx} - F_i - F_x + \rho^m [\lambda v_i + (1 - \lambda) v_{xm}] + \rho^x [\lambda v_{ix} + (1 - \lambda) v_x],$$

that is, they substitute away from mobile calls whenever possible and mobile-to-mobile calls only have weight  $\lambda$ .

The corresponding profits become:

$$\begin{aligned} \pi_i = & \mu_i \{F_i - f\} + \rho_i^m [\rho^m (p_i - c) - \rho_j^m n] q_i \\ & + \rho_i^m \rho^x (p_{ix} - c_o - a_x) q_{ix} + \rho_i^m n [\rho_j^m q_j + \rho^x q_{xm}]. \end{aligned}$$

Compared to (7), the weights of mobile-to-mobile incoming and outgoing calls has de-

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<sup>11</sup>On the other hand, the relative ranking of  $p_{ix}$  and  $p_{xm}$  is irrelevant; when a customer is at home and contemplates making a fixed-to-mobile call then he also has fixed-to-fixed calls at price  $p_x < p_{ix}$  at his disposal and thus will not be using his mobile phone in any case.

creased, while now there are more incoming fixed-to-mobile calls. Following the same approach as in the proof of Proposition 1, the following result is found:<sup>12</sup>

**Proposition 2** *Under Assumption 2, symmetric mobile equilibrium tariffs are given by:*

1. *a mobile-to-mobile call price at or below marginal cost,*

$$p_i^* = c + \frac{n}{2} + n (q_i/q_i') \Omega^{low}; \quad (14)$$

2. *a mobile-to-fixed call price at or below marginal cost,*

$$p_{ix} = c_o + a_x + n (q_{ix}/q'_{ix}) \Omega^{low}, \quad (15)$$

where

$$\Omega^{low} = \frac{\rho^m q_i/2 + \rho^x q_{xm}}{1/(2\sigma\Phi) - \rho^m q_i^2/q'_i - \rho^x q_{ix}^2/q'_{ix}};$$

3. *fixed fees equal to*

$$F_i^* = f + \frac{1}{2\sigma} \left( 1 - n \frac{\rho^m \Omega^{low}}{\mu\Phi} \right).$$

4. *Equilibrium profits are*

$$\pi_i^* = \frac{\mu}{4\sigma}. \quad (16)$$

Proposition 2 presents an interesting novelty as compared to Proposition 1. Recall that in the previous case the presence of a high termination rate leads to expensive fixed-to-mobile calls, i.e.,  $p_{xm} > p_i$ . In this case there is no distortion in the mobile-to-mobile price  $p_i$  because both M-only and FM-customers make the same number of mobile-to-mobile calls. When instead the mobile termination rate is low, fixed-to-mobile calls become cheaper than mobile-to-mobile calls, i.e.,  $p_{xm} < p_i$ . In this case, the price of mobile-to-mobile calls,  $p_i$ , is downward distorted because FM-customers substitute away from these calls when they are at home. Thus, firms use both mobile-to-fixed and mobile-to-mobile calls to price discriminate.

Finally, the call prices and fixed fee on the fixed network are the same as above: In particular, the fixed fee does not change since call prices are set at cost and therefore

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<sup>12</sup>For reason of space the proof of this Proposition is not reported. It is available from the authors upon request.



even though the number of calls on the fixed network has increased this has no effect on profits.

### 3 Simulation Results

#### 3.1 The Effects of Termination Rates on Subscriptions

This section studies the effect of fixed and mobile termination rates on subscription decisions. In particular, the question is whether higher termination rates raise or lower the number of subscribers for each network technology, and whether there are cross-effects. Due to the complex expressions for the equilibrium fixed fees and mobile-to-fixed prices, and independently of the actual subscription demand model (6), numerical simulations must be performed.

In the following, a logit subscription demand model is adopted. Denote the outside option of not buying any subscription as  $k = o$ , with consumer number  $\mu^o = 1 - \mu^x - \mu^{mx} - \mu^m$ , surplus  $\bar{w}^o = 0$  and the set of options  $K = \{o, m, mx, x\}$ . Then, the sizes of customer segments are given by

$$\mu^k = \frac{\exp(b\bar{w}^k)}{\sum_{l \in K} \exp(b\bar{w}^l)}, \quad k \in K.$$

Here,  $b > 0$  measures the degree of heterogeneity in subscribers' tastes for the different types of subscriptions. The corresponding measure of consumer surplus is  $CS = \ln(\sum_{k \in K} \exp(b\bar{w}^k)) / b$ . It is further assumed that consumer mobility is  $\lambda = 0.5$ ; mobile market differentiation is  $\sigma = 1$ ; call demand is  $q(p) = 5(1 - p)$ , and marginal costs are  $c_o = c_t = 0.1$ ,  $c_{xo} = c_{xt} = 0.01$ . Fixed costs  $f$ ,  $f_x$  and  $A_x$  are normalized to zero, and  $A_m = A_{mx} = 1$ . Varying the termination rates  $c_{xt} \leq a_x < c_t$  and  $a \geq c_t$  (with a change in substitution pattern at  $a = a^* = 0.28$  as explained above),<sup>13</sup> one can determine how the number of fixed and mobile subscribers changes depending on the levels of fixed and mobile termination rates.<sup>14</sup>

Table 1 reports the results from simulating equilibrium subscriber shares. For different

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<sup>13</sup>All the following results correspond to the "high termination rate case" for  $a \geq a^*$  (including the border case at  $a^* = 0.28$ ), and the "low termination rate case" for  $a < a^*$ .

<sup>14</sup>The simulations have been run under other demand models and parameter configurations, with qualitatively similar results.

combinations of mobile and fixed termination rates  $a$  and  $a_x$ , it reports the total numbers of mobile ( $\mu_m + \mu_{mx}$ ) and fixed ( $\mu_x + \mu_{mx}$ ) subscribers, as well the number of those who do not subscribe to any offer ( $\mu^o$ ).

	Total Mobile			Total fixed			Non-subscribers		
$a \backslash a_x$	0.01	0.055	0.10	0.01	0.055	0.10	0.01	0.055	0.10
0.10	80.3%	79.5%	78.6%	57.7%	59.7%	61.4%	5.6%	5.6%	5.7%
0.19	81.8%	81.0%	80.2%	55.1%	57.2%	59.1%	5.6%	5.6%	5.6%
0.28	81.9%	81.2%	80.4%	53.3%	55.6%	57.6%	6.0%	6.0%	6.0%
0.37	82.7%	82.0%	81.2%	52.3%	54.7%	56.8%	6.1%	6.0%	6.0%
0.46	83.1%	82.4%	81.7%	51.6%	54.0%	56.2%	6.3%	6.2%	6.2%

**Table 1:** Effect of the levels of the mobile ( $a$ ) and fixed ( $a_x$ ) termination rates on the shares of mobile, fixed and non-subscribing customers.

Two main results are obtained. First, a higher fixed (resp., mobile) termination rate increases the total number of fixed (resp., mobile) subscribers. This is by virtue of the waterbed effect: Higher profits from the termination of cross-network calls are handed on to consumers through lower subscription fees. These lower subscription fees then increase the take-up of the respective service. On the other hand, increasing a termination rate decreases the total number of subscribers on the other network. For example, increasing the mobile termination rate makes some consumers drop their fixed connection and opt for only having a mobile one. This effect occurs because setting a higher mobile termination rate increases the price of fixed-to-mobile calls (via a "cost effect"), which makes keeping a fixed connection less attractive. These simulation results suggest that the regulatory policy in effect during the last decade increased fixed-mobile access substitution via two separate channels: The conjunction of higher mobile termination rates and low fixed termination rates both stimulated the take-up of mobile services, while encouraging consumers to drop their fixed connection.

The second main result is that the number of customers who subscribe to neither a fixed nor a mobile network increases with the level of the mobile termination rate (The effect of the fixed termination rate is ambiguous). While the simulations confirm the often-heard claim that higher mobile termination rates increase mobile uptake, they also

show that the accompanying reduction in the number of customers of the fixed network is larger – the pool of non-subscribers increases. As just mentioned, the latter effect is due to higher fixed-to-mobile prices and the resulting lower surplus from holding a fixed connection. Thus, society faces a trade-off between realizing more benefits from mobility, through a larger number of mobile subscriptions, and more connectivity, through a larger number of consumers who are connected to any network. This and related welfare trade-offs are discussed in the following section.

### 3.2 Optimal Termination Rates

As shown in the previous section, fixed and mobile termination rates have an effect on subscriber numbers. An important question is then how different combinations of termination rates affect different groups of customers and the firms, and which leads to the highest welfare. Table 2 shows consumer surplus, profits of mobile operators and total welfare at equilibrium tariffs, for the termination rate values of Table 1.

	Consumer Surplus			Mobile Profits			Welfare		
$a \backslash a_x$	0.01	0.055	0.10	0.01	0.055	0.10	0.01	0.055	0.10
0.10	2.86	2.85	2.84	0.201	0.199	0.197	3.26	3.25	3.23
0.19	2.87	2.87	2.87	0.204	0.203	0.201	3.28	3.28	3.27
0.28	2.80	2.80	2.80	0.205	0.203	0.201	3.21	3.21	3.20
0.37	2.79	2.80	2.80	0.207	0.205	0.203	3.20	3.21	3.20
0.46	2.77	2.78	2.78	0.208	0.206	0.204	3.19	3.19	3.19

**Table 2:** Effect of the levels of the mobile ( $a$ ) and fixed ( $a_x$ ) termination rates on consumer surplus, profits and total welfare.

Clearly, the effects of mobile and fixed termination rates are quite different. Higher mobile termination rates increase consumer surplus and welfare while they are below  $a^*$ ,<sup>15</sup> but decrease both when they are above  $a^*$ . The latter occurs because fixed-to-mobile calls have become so expensive that just fixed-only customers use them – which even weakens the waterbed effect identified above.

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<sup>15</sup>Using a fine grid of termination rate values it has been verified that this is indeed the case.

At the same time, mobile operators' profits continue to increase over the whole range. Mobile operators thus prefer mobile termination rates above the welfare-maximizing level, even taking into account the benefits from mobility. On the other hand, consumer surplus, mobile operator profits and welfare are highest with fixed termination rates at cost (unless mobile termination rates are very high), which implies that setting them at cost is both the right thing to do and uncontroversial.<sup>16</sup> While these simulated values are not the outcome of a calibrated model (and thus should not be taken literally), they indicate that in the initial phase of the market it was indeed a socially optimal policy to set a fixed termination rate at cost and to allow mobile termination rates significantly above marginal cost.

It is also a relevant exercise to decompose consumer surplus into its various components, as each customer group should be affected differently. The corresponding simulations results have been collected in Table 3.

	Mobile-only			Fixed-Mobile			Fixed-Only		
$a \backslash a_x$	0.01	0.055	0.10	0.01	0.055	0.10	0.01	0.055	0.10
0.10	1.88	1.82	1.76	2.05	2.08	2.09	0.72	0.77	0.82
0.19	1.95	1.90	1.84	2.03	2.06	2.08	0.72	0.77	0.83
0.28	1.92	1.87	1.81	1.93	1.97	2.00	0.54	0.61	0.67
0.37	1.92	1.87	1.82	1.91	1.96	1.99	0.53	0.61	0.67
0.46	1.91	1.86	1.81	1.88	1.93	1.97	0.53	0.61	0.67

**Table 3:** Effect of the levels of the mobile ( $a$ ) and fixed ( $a_x$ ) termination rates on surplus of mobile-only, fixed-mobile and fixed-only customers.

All consumer groups prefer mobile termination rates at or below  $a^*$ , but just mobile-only customers have a clear preference for a mobile termination rate above cost (actually, just below  $a^*$ ). While the latter clearly prefer cost-based fixed termination rates, both fixed-mobile and fixed-only customers are in favour of higher levels.

Thus, the interests of customer groups diverge, which may in particular affects those consumers at the margin of dropping out of the market altogether. This indicates that the optimal regulatory policy to be adopted in each case may depend on whether for

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<sup>16</sup>Remember that by definition the fixed network's profits are zero in this scenario.

social or other reasons the interests of various customer groups are weighted differently or not.

## 4 Conclusions

Mobile telephony has been a tremendous success in the last two decades, making large inroads into the fixed telephony market. Subscribers in many countries now use their mobiles more than their fixed lines, and quite often disconnect the latter. This paper presented a model that captures the substitution between fixed and mobile telephony at the subscription and call level, by taking into account consumer mobility (i.e., the fact that consumers are sometimes at home, and sometimes on the road). In a rather general framework with two competing mobile networks and a regulated fixed network, the paper has shown that call substitution affects retail pricing incentives, in particular when customer heterogeneity makes it worthwhile to discriminate between customers with different substitution possibilities.

Numerical simulations are then used to analyze the effects of fixed and mobile termination rates on the number of fixed and mobile users. It is shown that termination rates do have an effect on subscription substitution and fixed disconnection: A higher mobile (fixed) termination rate increases the number of mobile (fixed) users and lowers that of fixed (mobile) users, via the waterbed and cost effects, respectively. Higher termination rates also tend to increase the number of customers who do not adhere to any network, identifying a trade-off between the uptake of individual networks and total uptake.

Lastly, the analysis indicates that socially optimal fixed termination rates tend to be at cost, which is the level also preferred by both mobile operators and mobile-only customers (but not fixed-mobile and mobile-only customers). On the other hand, socially optimal mobile termination rates are found above marginal cost, but below the level where they trigger substitution away from fixed-to-mobile calls. Mobile operators prefer rates clearly above the social optimum, mobile-only customers at the social optimum and the other consumers prefer rates below the latter. Thus, regulation of mobile termination rates is justified even taking into account access and call substitution, but interests of different customer groups do not coincide.

Future research will contemplate the possibility of bundling among fixed and mobile

services as well as network-based price discrimination and menus of tariffs.

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## Appendix

**Proof of Proposition 1:** Mobile operator  $i$ 's profits are  $\pi_i = \mu_i T_1 + \rho_i^m T_2$ , with

$$\begin{aligned} T_1 &= F_i - f + [\rho^m (p_i - c) - \rho_j^m n] q_i, \\ T_2 &= \rho^x (p_{ix} - c_o - a_x) q_{ix} + \mu_j n q_j + (1 - \lambda) \mu^x n q_{xm}. \end{aligned}$$

The following derivatives are needed, all originating from  $\mu_i^k = y (w_i^k - w_j^k) \mu^k$ , for  $j \neq i$  and  $k \in \{m, mx\}$ :

$$\begin{aligned} \frac{\partial \mu_i^m}{\partial F_i} &= -\sigma \mu^m, \quad \frac{\partial \mu_i^m}{\partial p_i} = -\sigma \mu^m \rho^m q_i, \quad \frac{\partial \mu_i^m}{\partial p_{ix}} = -\sigma \mu^m \rho^x q_{ix}, \\ \frac{\partial \mu_i^{mx}}{\partial F_i} &= -\sigma \mu^{mx}, \quad \frac{\partial \mu_i^{mx}}{\partial p_i} = -\sigma \mu^{mx} \rho^m q_i, \quad \frac{\partial \mu_i^{mx}}{\partial p_{ix}} = -\sigma \lambda \mu^{mx} \rho^x q_{ix}. \end{aligned}$$

Thus, one obtains, with  $\rho^\lambda = \mu^m + \lambda^2 \mu^{mx}$ ,

$$\begin{aligned} \frac{\partial \mu_i}{\partial F_i} &= -\sigma \mu, \quad \frac{\partial \mu_i}{\partial p_i} = -\sigma \mu \rho^m q_i, \quad \frac{\partial \mu_i}{\partial p_{ix}} = -\sigma \rho^m \rho^x q_{ix}, \\ \frac{\partial \rho_i^m}{\partial F_i} &= -\sigma \rho^m, \quad \frac{\partial \rho_i^m}{\partial p_i} = -\sigma (\rho^m)^2 q_i, \quad \frac{\partial \rho_i^m}{\partial p_{ix}} = -\sigma \rho^\lambda \rho^x q_{ix}, \end{aligned}$$

while the derivatives of  $\mu_j$  and  $\rho_j^m$  are identical apart from having the opposite sign. The first-order conditions for maximizing profits  $\pi_i$  over  $p_i$ ,  $p_{ix}$  and  $F_i$ , can then be written



as follows, for symmetric  $\mu_i = \mu/2$  and  $\rho_i^m = \rho^m/2$ .<sup>17</sup> For  $F_i$ , the first-order condition is

$$\frac{\partial \pi_i}{\partial F_i} = \frac{\mu}{2} - \sigma(\mu T_1 + \rho^m T_2) = 0,$$

from which one obtains directly

$$\pi_i^* = \frac{\mu T_1 + \rho^m T_2}{2} = \frac{\mu}{4\sigma}.$$

For  $p_i$ , the first-order condition is

$$\frac{\partial \pi_i}{\partial p_i} = \left\{ \frac{\mu}{2} \left[ 1 + \left( p_i - c - \frac{n}{2} \right) \frac{q'_i}{q_i} \right] - \sigma(\mu T_1 + \rho^m T_2) \right\} \rho^m q_i = 0.$$

Combining these two first-order conditions leads directly to

$$p_i^* = c + \frac{n}{2}.$$

For  $p_{ix}$  one obtains

$$\begin{aligned} \frac{\partial \pi_i}{\partial p_{ix}} &= \left\{ \frac{\rho^m}{2} \left[ 1 + (p_{ix} - c_o - a_x) \frac{q'_{ix}}{q_{ix}} \right] - \sigma(\rho^m T_1 + \rho^\lambda T_2) \right\} \rho^x q_{ix} \\ &\quad - \sigma \frac{\rho^m \mu}{2} \Phi n q_i \rho^x q_{ix} = 0, \end{aligned}$$

with

$$\Phi = \frac{\rho^\lambda}{\rho^m} - \frac{\rho^m}{\mu} > 0,$$

where the latter inequality follows from  $\lambda < 1$  and  $\mu^m, \mu^{mx} > 0$ . Combining the conditions for  $p_{ix}$  and  $F_i$ , and using  $T_1 = F_i - f$ , leads to the following outcome:

$$p_{ix}^* = c_o + a_x + n (q_{ix}/q'_{ix}) \Omega^{high},$$

where

$$\Omega^{high} = \frac{\mu q_i + (1 - \lambda) \mu^x q_{xm}}{1 / (2\sigma\Phi) - \rho^x q_{ix}^2 / q'_{ix}}$$

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<sup>17</sup>We assume that the corresponding sufficient second-order conditions hold.

Solving the first-order condition for the fixed fee then yields

$$\begin{aligned}
F_i^* &= f + \frac{1}{2\sigma} - \frac{\rho^m}{\mu} T_2 \\
&= f + \frac{1}{2\sigma} \left( 1 - n \frac{\rho^m \Omega^{high}}{\mu \Phi} \right) + \frac{\rho^m}{2} n q_i. \quad \blacksquare
\end{aligned}$$